CLASSICAL MECHANICS

Prof M. Brouard *4 Lectures*

Summary

1. *Translational motion:* Position, velocity and acceleration; Vectors and their addition; Newton's 1st and 2nd laws of motion.
2. *Momentum and Energy Conservation:* Newton's 3rd Law and conservation of linear momentum; Work, kinetic, and potential energy; Conservation of energy.
3. *Rotational motion:* Systems of particles and centre-of-mass; Angular velocity; Moments of inertia and angular momentum; Centripetal force and acceleration; Conservation of angular momentum.
4. *Vibrational motion:* Simple harmonic oscillator; Energy conservation; Travelling harmonic waves; Principle of linear superposition; standing and resonant standing harmonic waves.

Recommended Texts

1. *'Foundations of Physics for Chemists'*, Oxford Chemistry Primer 93, G.A.D. Ritchie and D.S. Sivia, 2000.
2. *'The Feynman Lectures on Physics'*, Volume I, Addison-Wesley Publishing Company, Reading, Massachesetts, 1963.
3. *'University Physics'*, Harris Benson, John Wiley and Sons Inc., USA, 1991.
4. Numerous A'Level texts...

Problems

1. (a) What do you understand by the term *equations of motion*?

   (b) An electron, initially travelling with speed v_0 at an angle θ to the horizontal, enters a region of length L between two parallel, horizontal charged plates, where it experiences a constant vertical acceleration, α.

   You may assume that the electron beam is initially directed towards the negatively charged plate. Any effects due to gravity may be neglected.

   i. Write down the equations of motion for the electron.
   ii. Show that the electron will enter and exit the plates at the same vertical displacement if the vertical acceleration is given by

   $$\alpha = \frac{2v_0^2}{L}\sin\theta\cos\theta$$

   iii. Show that under these circumstances the electron will emerge from the plates at an angle $-\theta$ to the horizontal.

2. (a) State Newton's three laws of motion, and for each law give one example of motion that illustrates it.

   (b) What factors determine the *trajectory* of a particle in classical mechanics?

   (c) A ball-bearing of mass, m, initially at rest, falls vertically under the influence of gravity through a viscous fluid, which exerts a retarding force of Cv, where C is a constant, and v is the speed of the ball-bearing. Assume that the acceleration due to gravity is a constant, g.

   i. Make a sketch illustrating the forces acting on the ball-bearing and write down its equation of motion.
   ii. Show that, at time t (measured from the moment of release), the speed of the ball-bearing is described by the equation

   $$v = \frac{mg}{C}\left(1 - \mathrm{e}^{-Ct/m}\right)$$

   iii. Sketch the time dependence of the speed of the ball-bearing and determine its terminal speed. How would you calculate the time dependent position of the ball-bearing?

3. (a) Define the terms *linear momentum* and *kinetic energy*. How are the two quantities related? How is the change in kinetic energy related to the work done by a (conservative) force?

   (b) Define the term *potential energy*. How is the change in potential energy related to the work done by a (conservative) force? In the light of your answer to (a), what is the significance of this result?

   (c) The potential energy between two argon atoms varies with the interatomic separation, r, approximately according to the equation

$$V(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^{6} \right]$$

   with $\epsilon = 1.7 \times 10^{-21}$ J and $\sigma = 3.4 \times 10^{-10}$ m.

      i. Sketch the variation in potential energy as a function of argon atom separation.

      ii. How does the force exerted on the argon atoms vary with atomic separation? At what separations is the force between the atoms attractive and at what separations is it repulsive?

4. Two argon atoms (A and B) of mass $m = 6.6 \times 10^{-26}$ kg undergo a *head-on* collision. Atom A has an initial speed of $400\,\mathrm{m\,s^{-1}}$ while the second atom B is stationary. You may assume that the interaction potential is as described in question 3(c).

   (a) What is the distance of closest approach of the two argon atoms, r_0? [Use energy conservation and make the substitution $x = (\sigma/r)^6$.]

   (b) What is the speed of the two argon atoms at r_0?

   (c) What is the acceleration between the atoms at r_0?

   (d) An *elastic collision* is one which conserves kinetic energy (in addition to total energy and momentum, which are conserved in all collisions). The collision between two Ar atoms is elastic.

   Given this fact, what are the final velocities of the atoms after the collision?

5. (a) A particle of mass m undergoes uniform rotational motion at a constant angular frequency ω. The particle is held fixed at a constant radius α from the axis of rotation.

      i. Assume that the particle rotates in the xy plane, and at time $t = 0$ lies along the x axis. Write down expressions for the x and y coordinates of the particle as a function of time, t.

      ii. Show that the linear velocity of the particle, $\mathbf{v}$, has magnitude $\alpha\omega$ and is directed tangentially to the orbital motion (i.e. perpendicular to its position vector $\mathbf{r}$).

      iii. Show that the radial (centripetal) acceleration of the particle has magnitude $a_\mathrm{r} = \alpha\omega^2$ and is directed in the opposite direction to the position vector $\mathbf{r}$.

   (b) Define the term *angular momentum.*

   (c) What is the *moment of inertia*, I, of the particle described in part (a)?

   (d) Expressing your answers in terms of I, determine the angular momentum and the angular kinetic energy of the particle defined in part (a). How are the two quantities related?

6. The rigid rotor model for rotation of an *isolated* diatomic molecule comprises two point masses, m_1 and m_2, separated at a fixed bond length, r.

   (a) Explain why the rotational motion must occur about the an axis which passes through the centre-of-mass of the molecule. Why is the angular momentum of the molecule conserved?

   (b) Show that the moment of inertia for the molecule is given by

$$I = \mu r^2 \; ; \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

   (c) Assuming that both HCl and DCl have bond lengths, $r = 1.27 \times 10^{-10}$ m, evaluate their moments of inertia.

   (d) The average rotational kinetic energy of a molecule at temperature T is $k_\mathrm{B}T$, where k_B is the Boltzmann constant (this comes from the *equipartition principle*).

   Use this fact to determine the average angular velocity and the average angular momentum of HCl and DCl at 300 K. Comment on the results you obtain.

   [Take $m_\mathrm{Cl} = 35\,u$, $m_\mathrm{H} = 1\,u$, and $m_\mathrm{D} = 2\,u$, where u is the atomic mass unit.]

7. The following expression describes the time dependence of the displacement of a harmonic oscillator with force constant k and of effective mass μ from its rest position

$$x(t) = A\sin(\omega t + \phi) \qquad \text{where} \quad \omega = \sqrt{\frac{k}{\mu}}$$

ω is the angular frequency of the oscillator, and ϕ determines the phase of the oscillation.

(a) Use this expression to determine how the velocity, $v(t)$, of the oscillator varies with time.

(b) Using the above expression for $x(t)$, together with your result for $v(t)$ in part (a), show that the total energy, E, of a harmonic oscillator is conserved.

(c) The vibrations of a diatomic molecule can be described approximately using the harmonic oscillator model.

i. Show that for such a system the effective mass is given by the expression

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

where m_1 and m_2 are the masses of the two atoms.

ii. The angular frequencies of the vibrations in $H^{35}Cl$ and $D^{35}Cl$ are $5.634 \times 10^{14}\,\mathrm{rad\,s^{-1}}$ and $4.041 \times 10^{14}\,\mathrm{rad\,s^{-1}}$ respectively. Determine the force constants for the two molecules. Comment on your results.

iii. Using your results from part (c)i, sketch the variations of the potential energy, $V(x)$, and the force, $F(x)$, with displacement for $H^{35}Cl$ and $D^{35}Cl$.

[Take the masses of H, D and ^{35}Cl to be $1.008\,u$, $2.014\,u$ and $34.969\,u$.]

8. The y-axis displacement of a harmonic wave travelling in the $+x$ direction may be represented by the function

$$y(x,t) = A\,\sin(kx - \omega t + \phi)$$

where t is the time.

(a) i. Define the parameters A, k, ω and ϕ appearing in this equation.

ii. How are k and ω related to the *wavelength*, λ, and the *period*, T, of the wave?

iii. How far does the wave travel from time $t = 0$ to $t = T$? Show that the *wave velocity* is given by $v = f\lambda$, where the frequency $f = 1/T$.

(b) A *standing harmonic wave* may be represented by the equation

$$y(x,t) = B(t)\sin(kx + \phi)$$

where $B(t)$ is the time dependent amplitude of the wave.

i. With the help of sketches, illustrate the time dependence of the wave motion.

ii. Show that such a wave can be constructed from superposition of two counter-propagating harmonic waves of the same frequency, amplitude and phase, and derive an equation for $B(t)$.

[Note that $\sin X + \sin Y = 2\,\sin\frac{1}{2}(X+Y)\,\cos\frac{1}{2}(X-Y)$.]

iii. Show that a *resonant standing wave*, with fixed nodes at $x = 0$ and $x = L$, will generate harmonics at the following frequencies

$$f = \frac{nv}{2L} \qquad n = 1, 2, 3, \cdots$$